Helmholtz' Theory of Concordance

In 1857 Helmholtz gave a lecture to an audience in Bonn[1] in which he posed the following question:

> *It is an acknowledged fact that the numbers of vibrations of concordant tones bear to each other ratios expressible by small whole numbers. But why? What have the ratios of small whole numbers to do with concord?*

The simplest example of concord is the octave and two notes an octave apart have frequencies in the simplest possible ration namely 2:1. Likewise, the frequencies of notes a fifth apart involve a ratio of 3:2 etc.

Helmholtz then, rightly, proceeds to point out that the notes produced by musical instruments such as violins and trumpets etc. all contain overtones or harmonics which are all whole number multiples of the frequency of the fundamental. He also points out that the overtones of percussion instruments such as bells are not harmonic (i.e. they are not necessarily whole numbers of the fundamental) and that this is the reason (he claims) why these instruments do not sound as pleasant as the one with harmonic overtones.

There is a problem with this theory.

The ear is a very remarkable organ which basically consists of a long tube called the cochlea filled with sensitive hairs each tuned to a different frequency. When presented with a sound waveform it is able to instantly carry out what is known as a Fourier transform and basically what it sends to the brain is information about the frequency and intensity of all the overtones which make up the waveform. (Information about the phase if the overtones is largely ignored.). When the ear hears the sound of a trumpet playing a note whose fundamental frequency is f, hairs tuned to the frequencies f, $2f$, $3f$, $4f$ etc. are excited to a greater or lesser extent. If a violin plays the same note, the same pattern of hairs will be excited but the relative strengths will be different. In this way the brain can deduce not only what note the instrument is playing bu also what instrument is playing it[2]. If, however, the note is played by a bell which has the following sequence of overtones: f, $1.9f$, $2.7f$, $3.6f$, the brain will receive a different pattern and obviously the brain will recognize that the note is being played by a completely different instrument. But why should the violin sound more pleasing than the bell? In fact, who is to say that the sound of a violin is more pleasing than that of a bell anyway? All that the brain knows is that the pattern of excited hairs is different. There is absolutely nothing in the structure of the ear which can tell the brain that the hair which is excited by a frequency of $2f$ is in any way related to the hair which is excited by the frequency f. It is my opinion that if I had been brought up in Bali surrounded every day by the sound of the gamelan, I would regard the sound of a bell as highly concordant and would possibly recoil in horror at the sound of a violin!

In Helmholtz' view

> *The art of the bell-founder consists precisely in giving bells such a form that the deeper and stronger partial tones shall be in harmony with the fundamental tone as otherwise ther bell would be unmusical.*

Here Helmholtz has committed the classic error of assuming the fact which he is trying to prove (that harmonic intervals are concordant). As we have seen, there is no *a priori* reason why

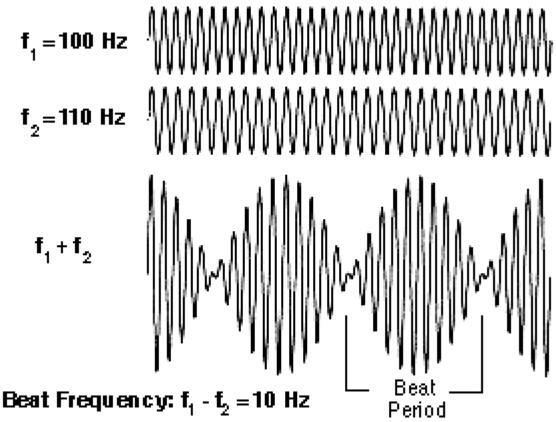
1 Reproduced in "Popular Scientific Lectures" published in translation by Longmans, Green, and Co in 1889 pp54 - 93

2 Actually there is a bit more to it than this. If you cut off the transients at the beginning of the note and just play the sustained section it can be surprisingly difficult to distinguish a violin from a trumpet.

harmonic intervals should necessarily be any more concordant than non-harmonic intervals.

Helmholtz then goes on to discuss another red herring – beats.

When two notes of slightly different frequency are sounded together – for example, when a note is struck on a badly tuned piano – the two waveforms move in and out of phase resulting in a waveform with a rapidly changing amplitude whose frequency is equal to the difference between the two original frequencies.



If the beat frequency is slow, this may manifest itself as a wow-wow sound, but when the beat frequency lies between 4 and 10 Hz, the sound becomes harsh and grating. The hairs in the cochlea which are excited are too close together to be distinguished by the brain, but the rapidly changing amplitude is detected. Only when the two frequencies are sufficiently far apart (about 15 Hz in the region of middle C) does the brain begin to perceive that two different notes are being played. Helmholtz then goes on to point out that if three notes of frequency $4f$, $5f$ and $6f$ (e.g. middle C, E and G on the piano) are struck simultaneously, then each pair of adjacent notes will generate beat frequencies of f. It is the concordance of these 'secondary tones', Helmholtz claims, which is the reason why the major triad sounds so pleasing.

There are two difficulties with this theory. In the first place, it is very difficult to hear these very low 'concordant' frequencies. If the two notes sounded are middle C and E, the beat frequency is the C two octaves below middle C. There are people who claim that they can actually hear this note distinctly but the majority of people are quite unable to hear it. In fact, the beat frequency only becomes audible as a distinct note if there is significant non-linearity in the channel. Old fashioned AM radio used this effect. The audio information is coded onto a carrier wave of much higher frequency by varying the amplitude of the signal. Alternatively, you could say that multiple carrier waves were being superimposed on each other generating beat frequencies in the audible spectrum. In order to retrieve the beat frequencies and reject the carrier wave, significant non-linearity is introduced into the circuit in the form of a diode. What this means is that it is only those people with defective hearing that can hear these 'secondary tones'! It seems very improbable that the pleasing sound of a major triad is in any way due to the concordance of these secondary tones.

The second objection to this theory is that the minor triad (whose frequencies are in the ratio 10:12:15) has secondary tones in the ratio of 2:3. What this means is that, if you could hear these tones they would not be coincident, they would be a fifth apart.

I think we must reject Helmholtz' theory of secondary tones outright.

There is, however, one further possibility which Helholtz does not discuss. We have mentioned that when the beat frequency between two close notes lies between about 4 and 10 Hz the brain cannot correctly interpret the sound. Could it be that the reason why some intervals sound discordant be because there exist beat frequencies between the harmonics of the two notes?

Probably the most discordant interval in music is the interval between C and F# whose freqencies are in the ratio of 1:√2. Here is a list of the frequencies of the first few harmonics:

C	256	512	768	1024	1280
F#	362	724	1086	1448	1810

The closest pair in this list is 32 Hz apart so it doesn't look as if we are going to find an explanation for the discordant nature of the Devil's Tritone in clashing harmonics.

So where does this leave us. At the very end of his lecture Helmholtz states:

...the determination of consonant intervals necessitates capabilities in the ear of feeling the upper partial tones and analysing the compound systems of waves into simple undulations according to Fourier's Theorem. It is entirely due to this theorem that the pitches of the upper partial tones of all serviceable musical tones must stand to the pitch of their fundamental tones in the ratios of the whole numbers to 1, and that consequently the ratios of pitches of concordant intervals must correspond with the smallest possible whole numbers.

Helmholtz is perfectly right in pointing out that the reason for the concordance of intervals to Western ears at least must be to do with the fact that the most common instruments used in classical western music are strings and pipes which have a rich set of harmonic overtones but Fourier's theorem has nothing to do with it. The reason why strings and pipes generate harmonic overtones (i.e. overtones whose frequencies are a whole number of the fundamental) is entirely physical and has nothing to do with Fourier's theorem. As I have said, bells generally have a strange set of overtones and they do not necessarily sound discordant to ears which have heard them from childhood.

So what then is the real link between harmonic overtones and concordance? Why do two notes an octave apart in some sense 'sound the same'? Why are the intervals of the fifth, the major third and the minor third so pleasing to our ears?

The answer, I believe, is this:

Two notes sounded by instruments rich in harmonics sound pleasing in respect of the degree to which they *share harmonics*.

Consider the octave. Here are the first few harmonics of middle C and the C an octave above it:

C_4	256	512	768	1024	1280	1536	1492	2048
C_5		512		1024		1536		2048

It is immediately obvious that every harmonic of the upper note (C_5) is included in those of the lower (C_4) and consequently it would be easy to mistake one note for the other. This is the reason why two notes an octave apart 'sound the same' – they share a lot of harmonics.

Two nhjotes a fifth apart have frequencies in the ratio of 3:2. They will therefore share every harmonic which is a multiple of 6 times the common fundamental, i.e those with frequency 6, 12, 18 etc.

The major third is defined by the ration 5:4 and will share harmonics whose frequencies are 20 times the common fundamental and the minor third (6:5) with frequencies 30 times the common fundamental. Chords made of octaves, fifths, fourths, major and minor thirds will all share many harmonic frequencies but chords containing seconds, sevenths or tritones will not share many frequencies and the brain will find it much more difficult to sort out which of the cacophony of frequencies belongs to which note and consequently will find it more discordant.

I can think of one objection to this theory but it turns out not to be so much a refutation but a confirmation of the idea and it highlights an important feature of the theory which I have not yet mentioned.

A sine wave generator produces a note which has no harmonics at all. It is a pure sine wave. According to the theory suggested above, two notes sounded by two such generators should have no harmonics in common and should therefore sound discordant even if the interval is a concordant one such as a fifth or a major third.

Well the first thing to say is that the bare fifth produced by two sine wave generators can hardly be described as harmonious. It just sounds like two unrelated notes and no more or less harmonious than any other pair of notes chosen at random. That is why, in my definition of the theory I specified that the notes in question should be played by instruments *rich in harmonics*.

The second thing to say is that, while a major triad played by three sine wave generators may be said to sound quite harmonious to western ears at least, I believe that this is only because we have heard so many triads played on strings and pipes that we have *learned* to regard them as harmonious. I very much doubt that my hypothetical Balinese child would be so impressed.

Thirdly, ever since equal temperament became popular it is no longer even true to say that the frequencies of concordant intervals are in whole number ratios. They may be close but they are not exact.

In short, what Helmholtz has left out from his analysis of why certain intervals sound concordant and others less so is the role of the brain. Our brains have a huge capacity for adapting and learning. It is quite astonishing to think that a trained musician can hear a wrong note played by the second clarinet while the whole orchestra is playing – but this ability has to be learned. The ability to sing a tune in tune is not innate. It too has to be learned. A child surrounded by the sounds of strings and pipes will learn to find concord in notes which share a lot of harmonics. A child brought up in a culture full of the sounds of drums and bells will learn to hear sounds in a totally different way. The true answer to the question of why notes an octave apart 'sound the same' and why minor triads sound sad lies not in the physics of sound wave or the mathematics of Fourier Transforms – it lies in our brains.

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